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MODELING MIGRATION UNDER UNCERTAINTY

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Abstract. This article develops a model explaining migration from one region to another taking into account uncertainty about earnings.

This approach differs from that of human capital in that it introduces uncertainty with the adoption of an utility function which the potential migrant seeks to maximize.

The migration model is derived by using the idea of "optimal allocation" from the portfolio analysis.

INTRODUCTION

The purpose of this article is to develop a model explaining migration from one region to another, which will take into account uncertainty about earnings. According to this model migration is treated as an investment decision of an individual who seeks to maximize his welfare in conditions of uncertainty.

This approach differs from that of human capital in that it explicitly introduces risk as an element specified in the objective function faced by a potential migrant.

In the context of human capital theory as decision theory, migration may be viewed in a cost returns framework so that for an individual to migrate his expectations must be that the costs, money and non – money of immigration are equal, or less than, the difference in the present discounted values of the streams of benefits in the source and receiving areas.

The present model introduces uncertainty with the adoption of an utility function which the potential migrant seeks to maximize. Maximization of the aggregate utility function concerning all potential migrants by using the idea of “optimal allocation” yields a stochastic migration function.

Since, the model treats migration in an investment decision context we proceed with the presentation of models of migration as an investment in Human Capital. After, the model of migration under uncertainty will be presented. Thus, one may easily compare the new model with the previous owner and point out its main characteristics and advantages in explaining migration as an investment decision.

MODELS OF MIGRATION AS AN INVESTMENT IN HUMAN CAPITAL

Within this theory migration is viewed in a cost – and – returns framework, such that for an individual to migrate his expectations must be that the costs of migration are equal or less than the difference in the present discount values of the streams of benefits in source and receiving areas.

We may write migration as a function of:

$$m_j(t) = F \left[\frac{V_j(t) - V_i(t)}{V_i(t)} \right], F' > 0$$

where:

- $m_{ij}(t)$: labour migration from area (i) to area (j) in period (t).
 $V_j(t)$: discounted present value of the expected real income stream over a worker's planning horizon in area (j).
 $V_i(t)$: discounted present value of the expected real income stream over a worker's planning horizon in area (i).

Migration takes place as a result of individuals seeking to maximize their utility which is functionally related to the expected present value of income (pecuniary and psychic), the discount period usually taken to be migrant's working lifetime.

It is obvious that a study of the determinants of migration based on the human capital approach requires panel data of a certain sub-group of the population in question, and a number of seriously criticised assumptions (experimentally determined discount rates, the subjective role of time preference, upward adjustment role of income streams, the role of risk and uncertainty) which sometimes do not agree with real world and influence to a high degree the result of the estimations (Bowles, 1970; Gallaway, 1969; Greenwood, 1975; Navratil and Doyle, 1977; Garcia Ferrer, 1982; Mueller, 1981).

The difficulties of empirical estimation due to the above weaknesses led researchers, to alter the model in order to avoid criticisms.

We may define the discounted present value of the unexpected real income stream over a worker's planning horizon to include the employment opportunity element.

$$V_{i(0)} = \int_0^n P_{i(t)} Y_{i(t)} e^{-rt} dt, \text{ where:}$$

- $Y_i(t)$: net real income in area (i) in period (t).
 $P_i(t)$: probability of having a job in area (i) in period (t).
 r : discount rate, we also define:
 $P_i(t) = (E_i / L_i) (t)$

Where:

- E_i = the existing employment labour force,
 L_i = the existing total labour force in area (i).

$$V_{j(0)} = \int_0^{\infty} P_{j(t)} Y_{j(t)} e^{-\rho t} dt - C(0), \text{ where:}$$

- $Y_j(t)$: net real income in area (j) in period (t).
 $P_j(t)$: probability of having a job in area (j) in period (t).
 $C(0)$: initial cost of migration
 $P_j(t) = (E_j / L_j)(t)$

If we assume, as did Harris and Todaro (1970), Laber and Chase (1971), Yap (1976) and Salvatore (1981) that real income differentials remain constant over time, the empirical estimation of the model can be simplified by dealing with a one-period time horizon.

$$m_{ij} = F[Y(t)], F' > 0$$

$$\text{where: } Y(t) = \left[\frac{E_j}{L_j}(t) Y_j(t) - \frac{E_i}{L_i}(t) Y_i(t) \right] / \frac{E_i}{L_i}(t) Y_i(t)$$

in order to initialise all the information provided by the two variables we take them separately as:

$$m_{ij}(t) = G[Y(t), E(t)], G'_y > 0, G'_E > 0,$$

$$\text{where: } Y(t) = \frac{Y_j(t) - Y_i(t)}{Y_i(t)} > 0 \quad \text{and} \quad E(t) = \frac{(E_j / L_j) - (E_i / L_i)(t)}{(E_i / L_i)(t)} > 0$$

since unemployment rate $u = 1 - E/L$ testing the previous equation is equivalent to testing the equation:

$$m_{ij}(t) = H[Y(t), u(t)], H'_y > 0, H'_u > 0$$

$$\text{where: } u(t) = \frac{u_j(t) - u_i(t)}{u_i(t)} < 0$$

Since migration is a decision which is not reversible without implying considerable costs, it may be regarded as a function of the expected value of the explanatory variables. Assuming an adaptive expectations form, migration equation becomes:

$$m_{ij}(t) = f[Y(t), u(t), m_{ij}(t-1)]$$

By separating the "push" from the "pull" factors we extend the model into equation:

$$m_{ij}(t) = f[Y_i(t), Y_j(t), u_i(t), u_j(t), m_{ij}(t-1)]$$

MIGRATION UNDER UNCERTAINTY

The previous section gave as the opportunity to present the models explaining migration which have been based on the human capital theory.

In this part migration is treated as an investment decision but the approach differs from the human capital approach in that it explicitly introduces risk as an element specified in the objective function faced by a potential migrant (Langley, 1974). After the introduction of uncertainty (risk coefficient) in the objective function of the potential migrant, the migration model is derived by using the idea of "optimal allocation" from the portfolio analysis (Tobin, 1958).

We assume that the individual has one period utility function of the form:

$$U = a - ce^{-b\delta}$$

U is the utility derived from migration; a, b and c are parameters c, b > 0, a < 0 and δ is the net return from migration in the period under consideration. The net returns is a stochastic variable since there is uncertainty about both costs and returns of migration (defined to include both monetary and physic components) (Smith, 1979; David, 1974). We assume that the probability distribution of returns, which may represent the belief that a particular outcome will occur – which may be evaluated by observed actions or the migratory experience of other individuals or from the past migratory experience of the individual himself, is normally distributed.

$$\delta \sim N(\mu_\delta, \sigma_\delta^2)$$

We assume that the migrant maximizes the expected value of utility which given the normality assumption, is: (Farrar, 1962).

$$E(U) = a - c \left[\exp\left(-\frac{b}{2}\mu_\delta + \left(\frac{b}{2}\right)^2 \sigma_\delta^2\right) \right]$$

$$\max E(U) = \max \left[\mu_\delta - \frac{b}{2} \sigma_\delta^2 \right]$$

parameter b may be interpreted as a coefficient of risk aversion since risk aversion coefficients (Arrow, 1971; Pratt, 1964) defined by Arrow and Pratt for a given utility function (U)

are $R = -UU''/U'$ relative risk aversion coefficient

and $R_A = -U''/U'$ absolute risk aversion coefficient

In our case for $U = a - ce^{by}$ the relative risk aversion coefficient is $R = b > 0$. For an interpretation of risk aversion, consider an individual faced with a random income Y and offered the alternative of a certain income Y_0 . A risk averter would be willing to accept a value of Y_0 less than the mean value $E(Y)$ of the random income, the difference may be thought as an insurance premium. For $R = 0$ we have risk neutrality.

We may now proceed in our application of the "optimal allocation" idea by using the previous analysis as a background. We consider a region (A) with total population (T). Out-migration from the region to the rest of the world (faced by the potential migrant as a region (B) is not obstructed by any legal constitutions. Each household tries to improve standard of living and it is possible a number of people decide to migrate to other region(s). One may seek employment if there is unemployment in region A, or a better job (promotion) and/or higher earnings.

Each household faces two labour markets (in region A and outside the region) with different distributions of wages, and has a utility function $U(Y) = a - ce^{bY}$ where $Y =$ income. There are also monetary (transport costs, lost wages, seniority or pension rights) and physic costs to be considered before the decision for migration is to be taken. Any non-labour sources of pecuniary income at its disposal are assumed to be perfectly transportable and therefore will not be affected by a change in place of residence. Thus, in each time period it is possible that a percentage of the total population (T), $\lambda_1 T_t = L_{1t}$ will migrate to another region and the rest of the population $\lambda_2 T_t = L_{2t}$ remain in its homeland so that:

$$L_{1t} + L_{2t} = T$$

Each household (individual) functions in order to maximize its utility function. Looking at the total population (T) we aim to maximize the aggregate utility function $V(Y)_t$, where:

$$Y_t = Y_{1t} + Y_{2t} = L_1 w_{1t} + L_2 w_{2t}$$

where: Y_t = income of total population at time (t)
 Y_1 = income of migrants at time (t)
 Y_2 = income of the rest of the population remaining at A in time (t)
 w_1 = wages at the labour market outside region (A)
 w_2 = wages prevailing in the labour market of region A.

Since the components w_1, w_2 are assumed to be random variables normally distributed, utility maximization becomes equivalent to maximize the expected utility $E[V(Y)]_t$ or

$$\max V(Y)_t \sim [E(Y) - 1/2R \text{var}(Y)]_t = G$$

where: $E(Y) = \bar{Y}_t$, $\text{var}(Y_t) = \text{variance of } Y$ and $R = \text{coefficient of relative risk}$

aversion ($R = b$ for our utility function). We also have

$$E(Y) = \bar{w}_1 L_1 + \bar{w}_2 L_2$$

$$\text{and } \text{var}(Y) = \sigma_{w_1}^2 L_1^2 + \sigma_{w_2}^2 L_2^2 + 2L_1 L_2 \text{Cov}(w_1, w_2)$$

where: \bar{w}_1, \bar{w}_2 are average wages.

Taking the first derivatives for L_1, L_2 ($\partial G/\partial L_1, \partial G/\partial L_2$) equal to zero we have:

$$\frac{\partial G}{\partial L_1} = \bar{w}_1 - R\sigma_{w_1}^2 L_1 - RL_2 \text{Cov}(w_1, w_2) = 0 \quad (\text{a})$$

$$\frac{\partial G}{\partial L_2} = \bar{w}_2 - R\sigma_{w_2}^2 L_2 - RL_1 \text{Cov}(w_1, w_2) = 0 \quad (\text{b})$$

Solving the system we obtain the solution for L_1 :

$$L_{1t} = \frac{\bar{w}_{2t} \text{Cov}(w_1, w_2)}{R[\text{Cov}(w_1, w_2)]^2 - \sigma_{w_1}^2 \sigma_{w_2}^2} - \frac{\bar{w}_{1t} \sigma_{w_2}^2}{R[\text{Cov}(w_1, w_2)]^2 - \sigma_{w_1}^2 \sigma_{w_2}^2}$$

The above equation determines the number of migrants at each time period (t). We may write it as $m_t = Aw_{2t} - Bw_{1t}$ where $m_t = \text{number of migrants at time period (t)}$. The signs of coefficient A, B depends upon $\text{cov}(w_1, w_2)$ since:

$$(\pm 0) \quad (+)$$

$$L_{1t} = \frac{\bar{w}_{2t} \text{Cov}(w_1, w_2)}{R[\text{Cov}(w_1, w_2)]^2 - \sigma_{w_1}^2 \sigma_{w_2}^2} - \frac{\bar{w}_{1t} \sigma_{w_2}^2}{R[\text{Cov}(w_1, w_2)]^2 - \sigma_{w_1}^2 \sigma_{w_2}^2}$$

$(+) \quad (+)(0) \quad (+) \quad (+) \quad (+)(0) \quad (+)$

$$\text{if } \text{Cov}(w_1, w_2) > 0 \Rightarrow L_{1t} = -\bar{w}_{2t}A + \bar{w}_{1t}B$$

$$\text{if } \text{Cov}(w_1, w_2) < 0 \Rightarrow L_{1t} = \bar{w}_{2t}A + \bar{w}_{1t}B$$

$$\text{if } \text{Cov}(w_1, w_2) = 0 \Rightarrow L_{1t} = \bar{w}_{1t} / \sigma_{w_1}^2 \sigma_{w_2}^2$$

Thus we have developed a migration model where out-migration depends on the average wages in two labour markets. To complete the model we introduce unemployment variables (which may consist of pull or push factors) additionally, since they influence

the decision and they are assumed to be constant for the short-time period (one year) and not random variables. Migration may be expressed as:

$$m_{ij,t} = f(\bar{w}_i, \bar{w}_j, U_i, U_j)$$

The derived model does take into account uncertainty about earnings in different locations, introducing the relative risk aversion coefficient. If $R=0$ utility maximization reduces to $\max G = E(Y)$. It may also apply for a number of different potential destinations $j > 2$. In this case migration (m_i) becomes a function of all "incomes" prevailing at the j locations ($j=1, 2, \dots, J$) (Wadycki, 1979).

For the case of three locations, migration to location (1) depends also on the income (average) prevailing in location (3) and the covariance of w_1, w_3 . Thus under the "optimal allocation rule" the decision making is also taking into account income opportunities in all potential destinations. (Xideas, 1987).

CONCLUSIONS

The migration model just derived, considers uncertainty about earnings in different locations by introducing explicitly the relative risk aversion coefficient. This constitutes main difference from the human capital model of migration which like a human capital theory implicitly assumes risk neutrality in the behavior of migrants. This assumption constitutes one of the major criticisms against human capital theory.

The role of risk and uncertainty affects migrants' decision making, since it is a function of the prevailing macro-economic climate. One would expect different behavior from potential migrants regarding the adoption of risk-averse behavior or not. The introduction of the utility function facing potential migrants leads to the inclusion of risk coefficients in the objective function under maximization.

Another crucial feature of this model deriving from the application of the portfolio theory is that migration to a location is also depended on the economic conditions (income-employment) prevailing to the rest of locations available for someone to migrate to.

This constitutes an improvement of the existing models of migrations since its explicitly takes onto account not only economic conditions prevailing in the two places of origin and final destination but also in all the available places of destination under consideration.

These two features contribute to the improvement of migration modeling. Firstly, they permit the adoption of an assumption concerning the behavior of migrants regarding risk and uncertainty. Secondly, they result to the conclusion of variables reflecting economic conditions existing to all possible destinations as well as to the consideration of all the existing correlations among them.

Thus, the maximization of migrants' utility function as well as the application of the idea of "optimal allocation" provide an improved model compared to previous models based on the human capital or other theories.

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