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## **A decision theory approach to determine a capital joint venture structure: a manager's view\***

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### **Abstract**

The literature on capital joint venture is reviewed to suggest a decision theory framework by which a single company may decide on its capital share in developing a new project. The focus is on the manager's view of the capital joint venture as a risk reduction means of his own. Concepts from reliability theory were borrowed to analyze the optimal share and the factors influencing it under an exponential utility form. Further, the probability distribution of the new project was contaminated with a probability mass for a negative net gain, designating failure of the project. Both analytical and numerical results show, as expected, that portfolio diversification is desirable for a risk averse manager, but the optimal ownership ratio of the new project is quite insensitive to correlation of the existing and new projects. On the other hand, it is very sensitive to the probability and the cost of failure of the new project. For comparison, the analysis is repeated with underlying Normal distribution. The lower risk under the Normal distribution determines a higher optimal share in the joint venture. The model was used to support the joint venture agreement of Dead Sea Works, Ltd. with Volkswagen Inc. for a new magnesium production facility at the Dead Sea in Israel.

Keywords: Joint Venture, vNM Utility, Bivariate Exponential Distribution

### **1. Introduction**

The purpose of this work was to determine the desired level of undiversified risk a manager is willing to bear in a joint venture. In the case presented, a manager must decide on his company's share in a new production plant, which is to be built in addition to an existing one. In contrast to stockholders, the manager's risk cannot be reduced by diversified portfolio; thus, his entire risk is essentially characterized by his corporate success. To establish a background, we first review the main interests of conglomerate managers in joint ventures, as described in the economics, financial, and international business literature (von Neumann and Morgenstern, 1953) (vNM). We then develop and solve a managerial expected utility model to determine the desired joint venture share from the manager's point of view.

The decision-making setting dealt with in this paper is based on observations of a large chemical company in Israel, The Dead Sea Works, Ltd. (DSW). The DSW extracts more than 2 million tons of potassium a year from the Dead Sea and exports most of it around the globe. The potassium is mostly used as an agricultural fertilizer, but it is also used for chemical, food, and other

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industries. The DSW, a public company, is owned by both the Government of Israel and private investors, but the government had designate it for privatization. Clearly, the short-term performance, as well as the long-term potential, of the DSW are crucial for this purpose.

Recently, the DSW Board of Directors decided to acquire the technology needed for extracting magnesium in addition to potassium. Magnesium is used by metal industries such as automotive, construction, and aircraft. Thus, the potassium and magnesium markets are usually not correlated, except possibly during a global market recession. The motivation for DSW's expansion stems from the economic advantage obtained by: (a) utilizing a raw material that is already available and partially processed by the potassium extraction process, (b) sharing logistical infra-structure with the existing products; and, (c) reducing individual industry marketing risks. However, such an extension involves acquisition and development of new technology and the construction of a new plant. Significant capital investment (in the hundreds of millions of U.S. dollars) is required for these purposes. Realizing the risk, the DSW manager considers proposing that the Board of Directors form a joint venture with a foreign investor to build the new plant. The joint venture not only serves the interests of the executive management, but also the government's aim to privatize the company. The Board of Directors is free to choose whether to form a joint venture and how to divide the share between the existing and the foreign owners. This was the main inquiry presented to the authors by the DSW management. According to the analysis in this paper, the optimal share should be between 0.544 and 0.62 (depending on the correlation between the magnesium and potassium markets). As a result, the final agreement was set at a share of 0.5 for both sides of the joint-venture.

Co-author Ehud Fuchs, who served as a Senior Vice President at DSW during the project represents the decision maker. As an economist, Dr. Fuchs identified the problem and bridged the gap between application and theory. He wanted to verify his intuition that partial ownership of the new facility might be an optimal policy in view of the technological risk and large investment involved. The magnesium project is a classic example of a cooperative international project that has been enabled by the Middle East peace process. The production technology is purchased from an eastern European company, Volkswagen, the joint venture partner is a western European company, and the facility is constructed and operated by the DSW, a Middle Eastern company that is searching for cooperative ties with Jordanian companies. Such cooperation was unthinkable just a few years ago. Clearly, the considered joint venture is not just a share of ownership, but also a vertical merging of producer and consumer of magnesium. Thus the benefits to both sides exceed the portfolio type risk reduction benefits. These additional benefits, however, are difficult to evaluate and beyond the scope of this paper.

The operational profitability of the new plant depends on both operational risks due to learning barriers of a new technology, and market risks related to price fluctuations and market instability. The operational profit generated by installing the new plant (hereafter termed "the new project") is possibly correlated with the operational profits generated by the company's existing products. Although these are different from the new product, they all belong to a "family" of chemical products. Thus, market risks characterizing the chemical industry in the economy at large are common to both the company's existing and new products.

The international business literature on joint ventures perceives international alliances as a major form of business aimed at increasing efficiency in global markets. Moreover, it is considered a significant challenge for multinational corporations to form corporate synergism by the formation of international joint ventures. The important question - to whom (stockholders or managers) a capital investment formed by a joint venture can be beneficial - is also discussed but not analyzed in this paper.

A major motivation of the joint venture is risk reduction, which can be analyzed as an insurance (see Kihlstrom and Pauly, 1971). Contractor and Lorange (1988) view synergistic effects of sharing information, technology, resources, markets, and risk as a type of economics of scale. While joint ventures can improve efficiency, they have also created problems for the partners because of different goals, values, and cultures. Geringer and Hebert (1991) claimed that the rate of success of joint ventures ranges from approximately 30% to 70%.

Joint venture performance is also determined by the amount of control exercised by each partner. Control is not a direct consequence of ownership. Control and ownership issues are discussed by Hennart (1988, 1991), Shenkar and Zeira (1992), Hu and Chen (1993), and others in light of the Transaction Cost Theory. The partners' interest is to minimize the costs associated with the coordination and conflict aspects of the joint venture. Factors such as the number of partners, the equity holding in the venture, the identity of the Board of Director's members, the cost from cooperation and coordination, dominate the control arrangement. Amit et al., (1994) analyze the relationship between venture capitalists and entrepreneurs with varying abilities. Our investigation concentrates on a Joint-Venture with a more symmetrical role of the partners. Both of them bring capital and a certain level of know-how, the asymmetric characteristic is the DSW ownership of extraction rights at the Dead-Sea.

Another factor that explains joint venture performance is identified by the socio-cultural gap between the foreign investors and the local partners in terms of values, management style, and business practice. Killing (1983) suggested that the smaller the cultural gap, the better the performance. Park and Russo (1996) perform an event history analysis of the electronic industry, they conclude that competitive relationship between the partners and concurrent agreements with others explain joint venture failures. The considered agreement is not characterized by these factors.

The financial literature on conglomerate mergers (including capital joint ventures) supports the view that risk reduction, through a capital joint venture diversification effect, will not be beneficial to stockholders, since they can achieve the desired level of risk on their own through market portfolio diversification (Levy and Sarnat, 1970; Amihud and Lev, 1981). This view holds even when market imperfections, such as transaction costs, are admitted. Given this view, a possible explanation for the conglomerate merger phenomenon, or, in particular, for the formation of capital by joint venture, is due to the manager's attitude toward risk. Risk-averse managers cannot diversify their own human risk in competitive markets. Thus, to stabilize the firm's annual income and to avoid the disastrous effects bankruptcy has on them, managers employ risk reduction means such as joint ventures.

Managerial risk reduction activities have been recognized in the literature (Jensen and Meckling, 1976; Harris and Raviv, 1978) as an agency cost. The manager (an agent), acting as an expected utility maximizer, may constitute a welfare loss to the stockholder (principal). The solution found in the financial literature (Harris and Raviv, 1978; Holmstrom, 1979; Shaveil, 1979; Fama, 1980) does not entirely eliminate the risk-reduction incentives of managers. Subsequently, we assume that the model under consideration is one that will maximize expected utility by means of risk diversification.

The economics literature on risk management considers two major types of risk reduction activities: (1) risk pooling, applied in the many-project case, regardless of the number of investors; and, (2) risk spreading, applied in the many-investor case, regardless of the number of projects. The argument put forth by Arrow and Lind (1970) with regard to risk spreading provides a justification for large organizations such as government to accept a project that might individually be rejected. In light of the previous discussion, which stems from the agency problem, this argument is unjustified in the setting suggested here. Moreover, several positions expressed in the early

economic literature regarding government attitudes toward pooling many risky projects (see, e.g., Arrow and Lind, 1970; Hirshleifer, 1965, 1966; Samuelson and Vickrey, 1964) seem inappropriate for the case presented in this study.

The purpose of our study is to complement this empirical literature by formulating and analyzing a normative managerial model that evaluates the benefits or the risk reduction gained by forming a capital joint venture. The setting suggested in the next section is concerned with a manager who maximizes his expected utility by determining the company's share in the capital invested in the new project. An extended model, analysis, and results are derived in Section 3, and final conclusions and future research directions are suggested in the last section.

## 2. The manager's expected utility model

Following the theoretical framework suggested by the vNM expected utility form, we formulate the decision maker's problem as a static one. As noted by Fishburn (1982), "von Neumann and Morgenstern offered their theory in a static or instantaneous mode. They attempted in their subsection 3.3.3 to avoid complications arising from preferences between events in different periods of the future." Karni and Schmeidler (1991) and Epstein (1990) provide more insight into the implication of this statement. Analyzing a non-static framework requires additional restrictions on the decision maker's preferences - restrictions that can only be modeled by nonlinear utility forms.

Let  $X$  denote the random net present value of future operating profits net investment attributed to a new project. The definition here of operating profit is sales minus production and logistic costs. This definition takes into account both direct and indirect costs. The net present value of the project initial investment (R&D and construction) denoted by  $K$  is assumed to be known. For a risk-averse vNM utility function denoted by  $U$ , the manager problem is formulated as follows:

$$\text{Max } E[U(\beta \cdot X)], \quad (P1)$$

$$\text{s.t. } 0 \leq \beta \leq 1$$

where  $\beta$  is the proportion of the project owned by the considered company. For illustrative purposes, we assume that the manager exhibits a constant positive risk aversion. That is:

$$U(W) = 1 - \exp(-W/a) \quad (1)$$

where  $1/a > 0$  is the constant measure of risk aversion as defined by Pratt (1964) and Arrow (1984), and  $W$  is the random monetary gain.

The exponential utility form has been employed in many studies (see, e.g., Brainard and Dolbear, 1971; Mehrez, 1985; Keeney and Raiffa, 1976). Let  $\beta^*$  denote the optimal solution of (P1). If  $E(X) > 0$  (i.e., the expected net present value of the project is positive), then, since  $U$  is a concave increasing function,  $\beta^* > 0$ . Further, if  $0 < \beta^* < 1$ , then it is unique and satisfies

$$E\left[\frac{X}{a} \exp(-\beta^* X/a)\right] = 0, \text{ since } U \text{ is a concave function in } \beta. \text{ Thus, it is easy to show by a}$$

comparative static analysis that  $\frac{\partial \beta^*}{\partial (1/a)} < 0$ , since  $\frac{\partial}{\partial (1/a)} E\left[\frac{X}{a} \exp(-\beta^* X/a)\right] < 0$  and

$$\frac{\partial}{\partial \beta^*} E\left[\frac{X}{a} \exp(-\beta^* X/a)\right] < 0.$$

Thus, the higher the risk aversion, as measured by  $(1/a)$ , the lower the  $\beta^*$ , or the optimal rate of the company's ownership in the joint venture. Furthermore, we see that a joint venture should not

be formed (i.e.,  $\beta^* = 1$ ) if  $E\left[\frac{X}{a^0} \exp\left(-\frac{X}{a^0}\right)\right] \geq 0$  and  $E(X) \geq 0$  for a given value of  $1/a^0$ . Clearly,

for  $1/a \leq 1/a^0$ , a joint venture is an inferior solution, since  $\frac{\partial \beta^*}{\partial (1/a)} < 0$  and  $U$  is concave.

As formerly mentioned, the DSW owns an existing successful production facility, and the new one (for which the joint-venture is considered) is being built in addition to that. The manager's decision might change if the company owns an existing income generator. He faces a constrained portfolio selection problem (see Konis 1996). Let  $Y$  denote the random present value of the annual operating profit over the planning horizon attributed to the existing system. Thus, (P1) can be further extended as follows:

$$\begin{aligned} \text{Max } E[U(Y + \beta X)], \\ \text{s.t. } 0 \leq \beta \leq 1 \end{aligned} \quad (\text{P2})$$

The introduction of a new product typically involves some technological and marketing risks, which can lead to the termination of the new project after the investment, but before any significant revenues are realized. In order to incorporate this phenomena in the model, we further assume that, with probability  $(1 - P)$ , the new project succeeds, but with probability  $P$ , it fails. If it succeeds,  $X$  and  $Y$  follow a certain bivariate probability distribution; but if it fails,  $X = -K$  for some fixed  $K > 0$ , representing the huge initial investment in the new plant. The marginal probability distribution of  $Y$  is identical under both situations. Therefore, the expected utility now becomes:

$$E[U(Y + \beta X)] = (1 - P)E[U(Y + \beta X)|X \geq 0] + P E[U(Y - \beta K)] \quad (2)$$

If  $X$  and  $Y$  are independent random variables and  $K$  or  $P$  is zero, then the analysis derived for (P1) still holds, and the determination of  $\beta^*$  is independent of the marginal probability distribution of  $Y$ . However, these restrictive assumptions rarely represent reality. In the next two sections, we formulate and analyze (P2) for a particular bivariate probability distribution.

### 3. The bivariate exponential distribution

We begin by evaluating the joint probability distribution function of  $X$  and  $Y$  (when  $X \geq 0$ ) by a modified bivariate exponential distribution function (MBEDF). The MBEDF has been originally implemented in statistical reliability applications (Barlow and Proschan, 1975). To the best of our knowledge, it has not been considered in economics decision theory settings. We propose an extension of this distribution to evaluate statistical dependency between random values of financial assets. Without loss of generality, we assume that  $X$  and  $Y$  are non-negative random variables.

Normal and Log-normal probability distributions are frequently used to represent financial revenues. We establish our choice of the MBEDF on two premises of the DSW's management: first, that under successful introduction of the new project, the present value of its future operational profits will exceed the initial investment; and second, under its failure, the new project will be terminated and no significant future profits will be gained. In addition, guided by the management's conservative attitude, we preferred to use a probability distribution (of the net present value of both the existing and the new facilities) with high density close to zero, large variability and skewed to the right. Another advantage of the MBEDF is its nonlinear correlation property. This property lets us define a random variable representing the global market state and impose a higher correlation between the two products under a "bad" global market state and a lower correlation under a "good" state.

### 3.1 The distribution function

The derivation of the EMBEDF stems from the assumption that, due to industrial and macro economics cyclical effects, the economy is represented by "good" and "bad" years. In general, the future performance of a particular project can be viewed as constrained by two independent forces: global market performance and product market performance. We write  $V \sim \text{Exp}(\lambda)$  to define  $V$  as a negative exponential random variable with  $E[V] = 1/\lambda$ . Let  $V_x \sim \text{Exp}(\lambda_x)$  denote the random net present value (NPV) of the operating profit within the manager's planning horizon, attributed to the new project under ideal (or "good") global market conditions. Similarly,  $V_y \sim \text{Exp}(\lambda_y)$  denotes the same for the existing plant. Let  $V_{x,y} \sim \text{Exp}(\lambda_{x,y})$  denote a global market performance measure. Suppose, in addition, that  $\alpha_x V_{x,y}$  represents a bound imposed by the global market on the performance of the new project. Then the NPV of the new project is defined by  $X = \text{Min}\{V_x, \alpha_x V_{x,y}\}$  and of the existing plant by  $Y = \text{Min}\{V_y, \alpha_y V_{x,y}\}$ .

Under "good" years, the forces that dominate the behavior of  $X$  and  $Y$  are unique for each product or sub-industry. However, an economic decline, identified by a low value of  $V_{x,y}$ , determines the random profit of  $X$  and  $Y$  via linear relationships (as a first approximation).  $\alpha_x$  and  $\alpha_y$  are positive linear coefficients relating  $V_{x,y}$  to  $X$  and  $Y$ , respectively. Notice that the original MBEDF is a special case of the EMBEDF, with  $\alpha_x = \alpha_y = 1$ . Here we allow for the effect of  $V_{x,y}$  on  $X$  and  $Y$  to vary according to the peculiarities of each product.

The larger  $\lambda_{x,y}$  is, the lower  $V_{x,y}$  tends to be, and (for given  $\alpha_x$  and  $\alpha_y$ ) the effect of the global economy on the industry economies increases. The lower  $\alpha_x$  is, the larger the effect of the global economy on the economy of  $X$ . Thus,  $\lambda_{x,y}$  can be interpreted as the rate of global events influencing the global economy, while  $\alpha_x$  measures the influence of such events on  $E(X)$ .

To evaluate  $\beta^*$ , we first summarize the main properties of the joint probability distribution function EMBEDF. The EMBEDF satisfies the following properties, which can be verified by employing probabilistic and algebraic operations ( $x$  and  $y$  denote realizations of  $X$  and  $Y$ , respectively). The principles of the properties derivations appear in the appendix.

**Property 1.** The conditional survival probability of  $Y$  on  $X$  is given by:

$$P(Y > y | X = x) = \begin{cases} \exp(-\lambda_y y), & \alpha_y x > \alpha_x y \\ \frac{\alpha_x \lambda_x}{\alpha_x \lambda_x + \lambda_{x,y}} \exp(\lambda_{x,y} x / \alpha_x - \lambda_y y - \lambda_{x,y} y / \alpha_y), & \alpha_y x \leq \alpha_x y \end{cases} \quad (3)$$

**Property 2.** The covariance of  $X$  and  $Y$  is given by:

$$\text{COV}(XY) = \frac{\alpha_x \alpha_y \lambda_{x,y}}{(\alpha_x \lambda_x + \lambda_{x,y})(\alpha_y \lambda_y + \lambda_{x,y})\lambda}, \quad (4)$$

where  $\lambda = (\alpha_x \lambda_x + \alpha_y \lambda_y + \lambda_{x,y})$ .

**Property 3.** The correlation coefficient of  $X$  and  $Y$  is given by:  $\rho(X, Y) = \lambda_{x,y} / \lambda$ .

**Property 4.** The regression function of  $Y$  on  $X$  is given by:

$$E(Y | X = x) = \frac{1}{\lambda_y} + \left[ \frac{\alpha_x \alpha_y \lambda_x}{(\alpha_x \lambda_x + \lambda_{x,y})(\alpha_y \lambda_y + \lambda_{x,y})} - \frac{1}{\lambda_y} \right] \exp\left(-\frac{\alpha_y}{\alpha_x} \lambda_y x\right). \quad (5)$$

**Property 5.**  $X$  and  $Y$  are exponential random variables with expected values and variances that are given by:

$$E(X) = \sqrt{\text{Var}(X)} = \frac{1}{\lambda_x + \lambda_{x,y}/\alpha_x} \text{ and } E(Y) = \sqrt{\text{Var}(Y)} = \frac{1}{\lambda_y + \lambda_{x,y}/\alpha_y}. \quad (6)$$

In the next section, we employ Properties 3, 4 and 5 for estimating the distribution parameters. Using this information, the optimal joint venture policy  $\beta^*$  for a given decision maker's utility function is then derived. It can be observed from Property 1 that the conditional cumulative probability distribution function is un-differentiable on the boundary of its two forms. It is interesting to note that  $\frac{\partial \rho(x,y)}{\partial \lambda_{x,y}} > 0$ ; thus, for  $\alpha_x, \alpha_y, \lambda_x$  and  $\lambda_y$  fixed, the sign of

$$\frac{\partial \beta^*}{\partial \rho(x,y)} = \frac{\partial \beta^*}{\partial \lambda_{x,y}} \frac{\partial \lambda_{x,y}}{\partial \rho(x,y)} \text{ is identical to the sign of } \frac{\partial \beta^*}{\partial \lambda_{x,y}}.$$

A neutral risk manager realizes an expected profit of  $U(\beta) = \beta E(X) + E(Y)$ . From Property 5, and the positive relationship between  $\lambda_{x,y}$  and  $\rho(X,Y)$ , it is implied that, for a given  $\beta$  in this case, the higher  $\rho(X,Y)$  is, the lower  $U(\beta)$ . Namely, the less diversified the investment, the lower the expected profit. Thus, unlike the traditional financial view, under the EMBEDF relativity law, diversification in production investment lowers the expected profit.

The effect of increasing  $\rho(X,Y)$  on the variance is inconclusive, since  $\text{Var}(\beta X + Y) = \beta^2 \text{Var}(X) + \text{Var}(Y) + 2\beta \rho(X,Y) \sigma(X) \sigma(Y)$ . Examples with less diversified investment can be generated to illustrate higher or lower variance for any fixed level of  $\beta$ . If, however,  $E(X)$  and  $E(Y)$  are fixed, then increasing  $\rho(X,Y)$  increases the variance of  $\beta X + Y$  without affecting its mean. This situation conforms with the traditional financial view that diversification reduces the portfolio variability and thus increases its expected profit.

### 3.2 Parameters estimation

The five probability distribution parameters of MBEDF ( $\alpha_x, \alpha_y, \lambda_x, \lambda_y$ , and  $\lambda_{x,y}$ ) are interconnected and should be estimated accordingly. We employ a combination of regression and moment estimation procedures, which results in consistent estimators for the considered parameters.

An ordinary use of the moment method would not be sufficient, since the marginal distributions of  $X$  and  $Y$  are exponential, thus having a single moment (only the first one) each, which can be used for estimation. Adding to that, their covariance provides an additional independent equation, or three together only (see Properties 3 and 5). Thus, in addition, we use the regression of  $Y$  on  $X$  (see equation 5), which has the following structure:

$$E(Y|X=x) = A + (C-A)e^{-Bx}, \quad (7)$$

where:

$$A = 1/\lambda_y, \quad (8)$$

$$B = \lambda_y \alpha_x / \alpha_y, \text{ and} \quad (9)$$

$$C = \frac{\alpha_x \alpha_y \lambda_x}{(\alpha_x \lambda_x + \lambda_{x,y})(\alpha_y \lambda_y + \lambda_{x,y})}. \quad (10)$$

We first use a numerical MSE (mean square error) estimation; namely, for a given sample  $(x_i, y_i)$ ,  $i = 1, K, n$ , we derive  $\hat{A}$ ,  $\hat{B}$ , and  $\hat{C}$  for (7) that minimize  $MSE = \sum_{i=1}^n (y_i - E(Y|X = x_i))^2$ .

Unfortunately, there is no unique local minimum in the region of  $A$ ,  $B$ , and  $C$ . However, this can be overcome by estimating  $\lambda_y$  beforehand, using historical data of  $V_y$ , and assigning  $\hat{A} = 1/\hat{\lambda}_y$ . Once  $A$  is estimated, the regression of  $Y$  on  $X$  can be transformed into a linear regression as follows:  $\ln(E(Y|X = x) - A) = \ln(C - A) - Bx$ . Next, we solve a system of equations, given by Properties 3 and 5 and by (7), (8), and (9), to derive estimates for the distribution parameters. In summary, the estimation process is as follows:

1. Estimate  $\lambda_y$  using  $\hat{\lambda}_y = 1/\bar{V}_y$ , where  $\bar{V}_y$  is the average of historical data of  $V_y$ , and set  $\hat{A} = 1/\hat{\lambda}_y$ .
2. Define  $y'_i = \ln(y_i - A)$  and  $C' = \ln(C - A)$ . Using a simple linear regression, estimate  $C'$  and  $B$ , and set  $\hat{C} = \exp(\hat{C}') + \hat{A}$ .
3. Estimate  $E(X)$ ,  $E(Y)$ , and  $COV(X, Y)$  from the data, and then solve for  $\alpha_x$ ,  $\alpha_y$ ,  $\lambda_x$  and  $\lambda_{x,y}$  the system of any four out of the five equations defined by Properties 3 and 5 and by (7), (8) and (9). The fifth equation can then be used for consistency verification.

### 3.3 Optimal policy

In this section, a closed form is derived for the objective function of (P2) as defined by (2). Its necessary optimality conditions, under the EMBEDF, are then provided. The objective function consists of two components. Following the derivation in the appendix, the first one, which stands for the expected utility under a success with the new project, is given by:

$$\begin{aligned}
 E[U(\beta, X, Y)|X \geq 0] &= 1 - E[\exp(-(\beta X + Y)/a)|X \geq 0] = \\
 &= 1 - E_X[\exp(-\beta X/a)E_Y[\exp(-Y/a)|X \geq 0]] = \\
 &= 1 - \frac{(\lambda_x \alpha_x + \lambda_{x,y})\lambda_y a}{(\lambda_y a + 1)(\lambda_x \alpha_x + \lambda_{x,y} + \beta \alpha_x/a)} - \\
 &\quad \frac{\lambda_{x,y}(a\lambda + \alpha_y)/(\alpha \alpha_y \lambda_y + a\lambda_{x,y} + \alpha_y)}{(\lambda_y a + 1)(\lambda_x \alpha_x + \lambda_{x,y} + \beta \alpha_x/a + (\lambda_y a + 1)\alpha_y/a)}. \tag{11}
 \end{aligned}$$

and the second component, which stands for the expected utility under a failure with the new project, is given by:

$$\begin{aligned}
 E[U(\beta, X, Y)|X = -K] &= 1 - E_Y[\exp((\beta K - Y)/a)] = \\
 &= 1 - \frac{\alpha \alpha_y \lambda_y + a\lambda_{x,y}}{\alpha \alpha_y \lambda_y + a\lambda_{x,y} + \alpha_y} \exp(\beta K/a). \tag{12}
 \end{aligned}$$

Problem (P2) can be solved for  $\beta$  by employing routine numerical analysis methods. As expected,  $\beta^*$ , which solves Problem (P2) with  $E[U(\beta, X, Y)]$ , as defined by (2), (11) and (12), satisfies that  $\frac{\partial \beta^*}{\partial P}$  and  $\frac{\partial \beta^*}{\partial K}$  possess negative signs. We are unable to determine analytically the sign

of  $\frac{\partial \beta^*}{\partial \rho}$  or equivalently of  $\frac{\partial \beta^*}{\partial \lambda_{x,y}}$ , but it will be investigated numerically in the next section. If  $P$  is sufficiently large, one would expect that  $\frac{\partial \beta^*}{\partial \rho} > 0$ ; namely, a higher correlation between  $X$  and  $Y$ , or less diversified economies, implies (given that  $X$  is non-negative) a higher value of  $\beta^*$  and a lower incentive for a joint venture capital structure. If  $P$  is sufficiently small, a counter example obtaining a negative sign for  $\frac{\partial \beta^*}{\partial \rho}$  can be detected. These results indicate that the willingness of the manager to bear risk or to increase his company's share in the joint venture may be higher if the likelihood of the failure of the new project is sufficiently low and the new and the existing projects are less diversified, given that both are profitable.

#### 4. The bivariate normal distribution

This section solves the same problem but with the assumption of bivariate normal distribution (BVN) replacing the MBEDF. Let  $X$ ,  $Y$ ,  $K$  and  $P$  be defined as before, where, under success,  $(X, Y) \sim BVN(\mu_x, \sigma_x^2, \mu_y, \sigma_y^2, \rho)$ , meaning that  $X$  and  $Y$  follow a bivariate normal distribution. Also, as before,  $Y$  (the net present value of the profit of the existing product) preserves its marginal distribution when  $X = -K$  under a failure of the new project. The manager's problem remains as defined by P2 with the same utility function (1). The objective function of P2 follows the same general structure as defined by (2). In the following, we specify the objective function, derive necessary conditions for optimality, and show that a single optimal solution satisfies these conditions.

Under the Bivariate Normal Distribution, the objective function of P2 becomes:

$$E(U(Y + \beta X)) = (1 - P) \left[ 1 - \exp \left( \frac{\beta \mu_x + \mu_y}{-a} + \frac{\beta^2 \sigma_x^2 + 2\beta \rho \sigma_x \sigma_y + \sigma_y^2}{2a^2} \right) \right] + P \left[ 1 - \exp \left( \frac{-\beta K + \mu_y}{-a} + \frac{\sigma_y^2}{2a^2} \right) \right]. \quad (13)$$

The derivation is given in the appendix. With this expression, it is easy to derive the first and second derivatives of (13) and use them to search for an optimal share  $b$  and to prove that the solution is unique. The following properties define the conditions for the interior solution.

**Property 6.** The conditions for the interior solution of  $b$  is given by:

$$\frac{PK}{a} \exp \left( \frac{K - \mu_y}{a} + \frac{\sigma_y^2}{2a^2} \right) > (1 - P) \left[ \frac{\mu_x}{a} - \frac{\sigma_x^2 + \rho \sigma_x \sigma_y}{a^2} \right] \exp \left( \frac{\mu_x + \mu_y}{-a} + \frac{\sigma_x^2 + 2\rho \sigma_x \sigma_y + \sigma_y^2}{2a^2} \right)$$

and:

$$\frac{PK}{a} < (1 - P) \left[ \frac{\mu_x}{a} - \frac{\rho \sigma_x \sigma_y}{a^2} \right]. \quad (14)$$

The proof is not provided for its simplicity. If the first condition is violated, then  $\beta^* = 1$ , and when the second one is violated,  $\beta^* = 0$ .

Contrary to the MBEDF case, under the BVN distribution, a special process for deriving the distribution parameters is not required.

When the installation of the new project involves no risk (i.e.  $P = 0$  or  $K = 0$ ) one can get a closed form expression for  $\beta^*$  as follows:  $\beta^* = \frac{a\mu_x - \rho\sigma_x\sigma_y}{\sigma_x^2}$ . Thus, in this special case, the share

the manager would invest in the new project decreases with the risk aversion ( $1/a$ ), and with the correlation of the two products  $r$ . With regard to conditions for a boundary solution, it follows that, when  $\sigma_{xy} \geq a\mu_x$ , then  $\beta^* \leq 0$ , and if  $\sigma_{xy} \leq a\mu_x - \sigma_x^2$ , then  $\beta^* \geq 1$ . Under the general case, Similar relationships between  $r$  and  $\beta^*$  prevail as demonstrated in the numerical example in the next section.

## 5. Numerical results

This section demonstrates the implementation of the method for the case of DSW. In addition to a detailed example comparing the two bivariate distribution functions, we present sensitivity analyses of  $\beta^*$  to the decision maker's risk factor  $a$  and to the correlation factor ( $r$ ) of  $X$  and  $Y$ . Some of the data have been altered in order to avoid exposure of proprietary information. Monetary figures are in Billion Dollars.

According to the business plan of the new facility, the following data can be determined: the expected annual return of the initial investment ( $K = 0.8$ ) and the probability of its failure ( $P = 0.1$ ). The decision maker's risk factor was estimated as being in the neighborhood of  $a = 1$ . Using expert estimates, we determined the expected profits:  $E(Y)$  of the existing product and  $E(X)$  of the new product. This data is required under both types of bivariate distributions. In the following, we first present the solution process under the EMBEDF and then under the BVN distribution.

### 5.1 Exponential distribution

Recall that, under the EMBEDF, the standard deviation of each random variable is identical to its mean, but other parameters should be estimated. First, the expected profits  $E(V_y)$  of the existing product under "good" global economy is estimated. Next, random data of  $X$  and  $Y$  is generated according to the marginal distribution of each. Due to lack of information regarding  $\rho(X, Y)$  and the distribution parameters of  $V_{x,y}$ , we have produced several matching sequences of  $X$  and  $Y$ , where each is characterized by a different value of  $r$ . Each such sequence represents a random sample of a certain EMBEDF with unknown parameters, but all with identical means of  $X$ ,  $Y$ , and  $V_y$ .

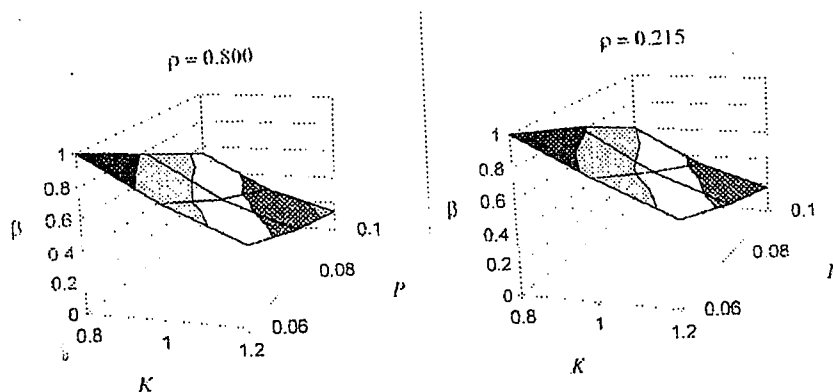
The problem was solved for each such sequence. First, the five probability distribution parameters ( $\alpha_x$ ,  $\alpha_y$ ,  $\lambda_x$ ,  $\lambda_y$ , and  $\lambda_{x,y}$ ) were estimated, and then  $\beta^*$ , the optimal joint venture policy, was derived. Table 1 presents the results for the random sample of each correlation level.

Table 1. Optimal solution and estimated parameters versus  $r$  under EMBEDF

$\rho$	$\beta^*$	$\hat{A}$	$\hat{B}$	$\hat{C}$	$\lambda_x$	$\lambda_y$	$\lambda_{x,y}$	$\alpha_x$	$\alpha_y$	$E(U)$
0.079	0.620	4.730	0.015	4.573	5.033	0.211	0.00034	0.00078	0.00006	0.1679
0.132	0.616	4.466	0.028	4.317	4.733	0.224	0.00059	0.00082	0.00010	0.1674
0.215	0.605	4.533	0.045	4.399	4.278	0.221	0.00073	0.00061	0.00013	0.1665
0.353	0.592	3.890	0.088	3.780	3.487	0.257	0.00089	0.00046	0.00016	0.1649
0.415	0.585	3.137	0.131	3.038	3.124	0.319	0.00094	0.00041	0.00017	0.1641
0.471	0.578	3.362	0.139	3.273	2.810	0.298	0.00098	0.00037	0.00018	0.1638
0.600	0.572	3.667	0.156	3.597	2.054	0.273	0.00107	0.00032	0.00019	0.1618
0.655	0.564	4.177	0.150	4.117	1.757	0.240	0.00110	0.00030	0.00019	0.1615
0.707	0.555	3.362	0.208	3.313	1.435	0.298	0.00113	0.00028	0.00020	0.1610
0.759	0.553	3.362	0.221	3.321	1.130	0.298	0.00115	0.00027	0.00020	0.1601
0.800	0.547	3.890	0.200	3.856	0.920	0.257	0.00117	0.00026	0.00021	0.1599
0.829	0.544	3.362	0.244	3.334	0.716	0.298	0.00118	0.00025	0.00021	0.1595

The main observations are summarized as follows:

1. As  $r$  increases,  $\beta^*$  decreases, but is not very sensitive to  $r$  in this example. We discussed this relationship before, but for when all other parameters are fixed. Here we observe this property for the more practical situation, with all the parameters at values consistent with the observed correlation  $r$  and fixed expected values of  $X$  and  $Y$ . This result is very important in the DSW case because of lack of data for estimating  $r$ .
2.  $E[U(\beta^*, X, Y)]$  decreases with  $r$ . This result agrees with the analytical arguments made previously, that, for fixed expected values of  $X$  and  $Y$ , the variance of the total profit increases with  $r$  and the expected utility of a risk averse decision maker decreases. However, the decrease is insignificant; again, insightful information for the decision maker.
3. In this example,  $\lambda_x$  is much more sensitive to  $r$  than  $\lambda_y$ . Recall that  $\lambda_y$  is estimated first using  $E(V_y)$ , and only then are the remaining parameters estimated in a coordinated manner. In our example, the expected value of  $V_y$  is not significantly influenced by  $r$ ; thus,  $\lambda_y = 1/E[V_y]$  only slightly changes with  $r$ . As a result, all the remaining parameters change significantly with  $r$ , especially  $\lambda_x$ . The increase in  $r$  involving an increase in  $\lambda_{x,y}$  leads to a decrease in  $\alpha_x$  and  $\lambda_x$ , such that  $E(X)$  is not changed.
4. The optimal policy  $\beta^*$  is much more sensitive to  $P$  and  $K$ . However, at least for the investigated case, the value of  $r$  does not significantly affect these sensitivities. Figure 1 presents two examples of  $\beta^*$  as functions of  $P$  and  $K$  for  $\rho = 0.8$  and  $\rho = 0.215$ . The differences between the two quite extreme values of  $r$  can hardly be recognized.

Figure 1.  $\beta^*$  as function of  $P$  and  $K$  under EMBEDF

## 5.2 Normal distribution

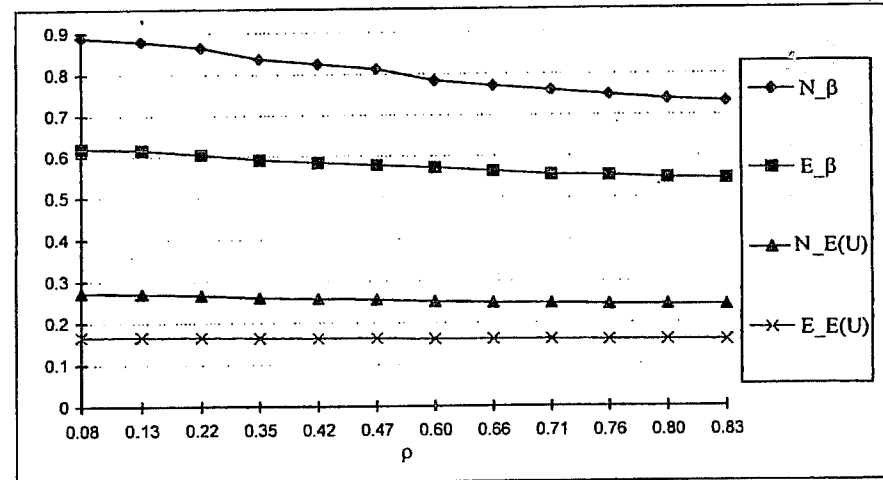
Under the BVN distribution, we face a much simpler estimation problem of the probability distribution parameters. One must estimate the standard deviation of each of the two random variables ( $X$  and  $Y$ ), in addition to their means. In order to provide a basis for comparison to the EMBEDF, same values of mean and standard deviation are used and similar values of  $\rho(X, Y)$  are examined. No further parameter estimation is required.

The optimal policy  $\beta^*$  and the corresponding expected utility value is provided in Table 2 for each correlation value of  $r$ . Figure 2 compares the optimal solutions and the corresponding expected utilities under both distributions. Note that  $E_-$  in the figure represents EMBEDF and  $N_-$  represents BVN.

Table 2. Optimal solution versus  $r$  under BVN distribution

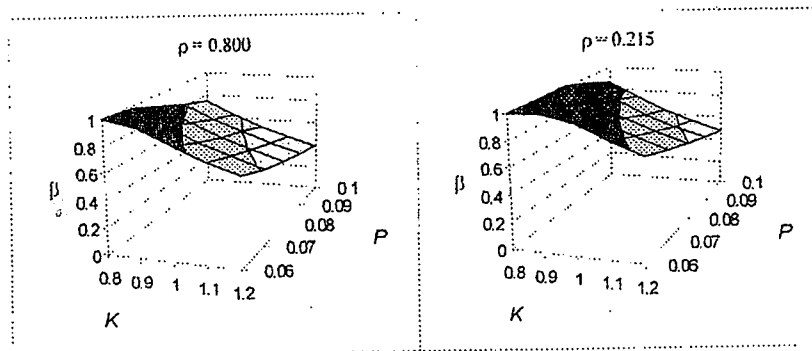
$\rho$	$\beta^*$	$E(U)$
0.079	0.889	0.2722
0.132	0.879	0.2700
0.215	0.864	0.2665
0.353	0.836	0.2608
0.415	0.824	0.2582
0.471	0.812	0.2560
0.600	0.784	0.2508
0.655	0.772	0.2486
0.707	0.761	0.2466
0.759	0.749	0.2446
0.800	0.739	0.2430
0.829	0.732	0.2419

Figure 2. Comparison of optimal solutions under BVN versus EMBEDF



The main observations are summarized as follows:

1. Similarly to the EMBEDF, as  $r$  increases, both  $\beta^*$  and  $E[U(\beta^*, X, Y)]$  decrease, but with a larger sensitivity to  $r$ .
2. Interestingly, the values of  $\beta^*$  are larger by 32% to 42% and  $E[U(\beta^*, X, Y)]$  by 50% to 65% under the BVN distribution versus the EMBEDF. A possible explanation is the larger risk involved under the exponential type distribution versus the normal one. A commonly used measure of the remaining variability is the Kurtosis. The Kurtosis of the single dimension exponential distribution is  $9/\lambda^4$ , which is three times larger than  $3\sigma^4$ , which is the Kurtosis of the single-dimension normal distribution with the same variance. Standardized measures of Kurtosis for double-dimension normal and exponential distributions are 3 and 6, respectively (see Mehrez and Stulman, 1982).
3. Figure 3 presents the two examples for the BVN distribution shown in Figure 1. The larger values of  $\beta^*$  under the BVN results in boundary solutions ( $\beta^* = 1$ ) for large values  $K$  and  $P$ . The marginal effect of  $K$  and  $P$  on the solution is about the same under both distributions.

Figure 3.  $\beta^*$  as function of  $P$  and  $K$  under BVN distribution

## 6. Conclusions and future research

An extension of the MBEDF is first applied here to describe economic profit behavior. Properties of this new bivariate statistical distribution and the estimation procedure for its parameters are provided as well. Its contamination with a discrete mass probability of a large negative value, to represent a technological or marketing failure with the new product, provides a meaningful structure and leads to insightful results.

The case of DSW joint venture with a large automotive manufacturer is analyzed. From the viewpoint of the DSW manager, the optimal joint-venture policy under the EMBEDF is more conservative than under the BVN distribution. This result raises the need for caution when the assumption of normal distribution regarding future profits is made.

Possible directions to extend the share are as follows:

1. Comparison of the numerical and theoretical results with a bivariate log-normal, or other types of distributions. In particular, comparisons might focus on the sign of  $\partial\beta^*/\partial\rho$ .
2. Analysis of other utility forms of the vNM type or of a non-linear type.
3. View of the problem from a game theoretic approach to determine procedures, rules for a set of efficient solutions, or the determination of the parties' shares in the joint venture.
4. Relaxation of the stationary assumption of the profit structure to allow for learning factors, as well as taking into account other dynamic changes.
5. The main purpose of this study was to introduce an alternative view of the economics behavior of random profits. Alternative shock models might be investigated to explore their effect on joint-venture decisions.
6. Consideration of pricing, as well as other marketing decisions, under stationary and non-stationary settings.
7. Improving the statistical theory aimed at estimating the EMBEDF model's parameters.

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## Appendix

### 1) Derivation of (11) and (12)

Let  $Z = \text{Exp}(-y/a)$  and  $z$  as its realization. Thus,  $P(Z > z | X = x) = P(Y \leq -a \ln(z) | X = x)$ , for  $0 \leq z \leq 1$ . Hence, by Property 1,

$$P(Z > z | X = x) = \begin{cases} 1 - z^{a\lambda_y}, & \text{Exp}\left(\frac{-\alpha_y x}{\alpha_x a}\right) < z \leq 1 \\ 1 - \frac{\alpha_x \lambda_x \text{Exp}(x\lambda_{x,y}/\alpha_x)}{\alpha_x \lambda_x + \lambda_{x,y}} z^{(\lambda_{x,y}/\alpha_y + \lambda_y)a}, & 0 \leq z \leq \text{Exp}\left(\frac{-\alpha_y x}{\alpha_x a}\right) \end{cases} \quad (\text{A1})$$

The conditional expectation of  $Z$  on  $X$  can now be evaluated as follows:

$$E[\text{Exp}(-y/a) | X = x] = \int_0^1 P(Z > z | X = x) dz. \quad (\text{A2})$$

Substituting (A1) into (A2) and simplifying gives:

$$E[\text{Exp}(-y/a) | X = x] = \frac{1}{\lambda_y a + 1} \left\{ \frac{\lambda_{x,y}(\lambda a + \alpha_y) \text{Exp}(-(\lambda_y a + 1)\alpha_y x / \alpha_x a)}{(\lambda_x \alpha_x + \lambda_{x,y})(\lambda_y \alpha_y + \lambda_{x,y})a + \alpha_y} + \lambda_y a \right\}. \quad (\text{A3})$$

**2) Derivation of (13)**

For the first part of (2), substitute  $Z = -(\beta X + Y)/a$  when  $Z \sim N(\mu_z, \sigma_z^2)$  with  $\mu_z = \frac{-1}{a}(\beta\mu_x + \mu_y)$  and  $\sigma_z^2 = \frac{1}{a^2}(\beta^2\sigma_x^2 + 2\beta\rho\sigma_x\sigma_y + \sigma_y^2)$ . Thus we obtain:

$$E[U(\beta, X, Y)] = 1 - E[\exp(-(\beta X + Y)/a)] = 1 - [\exp(Z)] \\ = 1 - \int \exp(Z)f(Z)dZ. \text{ Using the moment generation function of the normal distribution, the} \\ \text{expected utility becomes: } 1 - \exp\left(\mu_z + \frac{\sigma_z^2}{2}\right). \text{ The same derivation with } \mu_x = -K \text{ and } \sigma_x = 0 \\ \text{solves the second part of (2).}$$