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VALUE JUDGMENT VERSUS ALLOCATIVE EFFICIENCY: A CASE OF TENNESSEE COUNTY JAILS

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Abstract. Data envelopment analysis (DEA) is a proven mathematical programming approach for measuring the relative efficiencies and inefficiencies of decision making units (DMUs). The basic DEA models can be enhanced with standard techniques that allow incorporation of value judgments, e.g., assurance region (AR) and preference structure (PS), and thus provide estimates of allocative efficiency (AE) when the exact price information is not present. Using non-parametric statistical tests, the current study examines consistency and inconsistency among AE, DEA/AR and DEA/PS scores by measuring the productive efficiency of 33 Tennessee county jails. It is shown that both DEA/AR and DEA/PS scores are more correlated with a cost efficiency (CE) score which is a mix of a technical efficiency (TE) and an AE. It is also shown that by specifying a proper set of preference weights for each DMU, DEA/PS model gives the exact information on CE.

Keywords: Data envelopment analysis (DEA), assurance region (AR), preference structure (PS), allocative efficiency (AE)

1. INTRODUCTION

As described in Seiford (1996), since the first DEA (Data Envelopment Analysis) of Charnes *et al.* (1978), the evolution of DEA has been rapid and widespread resulting in a host of published articles. DEA was originally developed to measure the relative efficiencies and inefficiencies of not-for-profit DMUs when *a priori* information, e.g., market prices are not available. However, DEA can easily be extended to evaluate the (input) allocative efficiency (AE) when the (input) prices are present (see, e.g., Färe, Grosskopf and Lovell (1985)). As pointed out by Cooper, Thompson and Thrall (1996), AE can be of limited value in actual applications because it imposes severe data requirements and the prices can be (and often are) subject to variation over very short periods. Thus, an assurance region (AR) approach as proposed by Thompson *et al.* (1990) is frequently used instead of AE. (See, e.g., Sueyoshi (1992) and Zhu (1996a)). On the other hand, Zhu (1996b) develops a set of DEA models incorporating a preference structure (PS) where, by specifying a proper set of preference weights, DEA/PS scores can also be used to estimate the AE.

In other words, value judgments can be incorporated to estimate the AE when exact price information is not available. However, we should point out that the efficiency measured by DEA/AR or DEA/PS approach is not pure AE, since DEA/AR and DEA/PS mix technical and allocative efficiencies. AE is usually calculated from a cost efficiency (CE) measure which reflects the difference between (minimum) optimal cost and actual cost. In fact, technical efficiency (TE) and AE are two components of CE. Intuitively, DEA/AR or DEA/PS would be more related to CE than AE.

Although it has been noticed that the DEA/AR and DEA/PS score is not AE as defined in the economic literature, the test for inconsistency has not been empirically validated, i.e., applied in a real world setting. The purpose of the current paper is to examine the inconsistency and consistency of various measures related to AE by employing non-parametric statistical tests and data on 33 Tennessee county jails. As a result, the relationship between DEA/PS and CE is further revealed.

2. THE MODELS

In order to develop our discussion, we present several related DEA models (see Seiford and Thrall (1990) and Färe *et al.* (1994) for a detailed discussion of these DEA models). Suppose we have n DMUs. Each DMU_j, j = 1, 2, ..., n produces s different outputs, y_{rj} (r = 1, 2, ..., s), using m different inputs, x_{ij} (i = 1, 2, ..., m). Then the technical efficiency (TE) under constant returns to scale (CRS) can be calculated via the following DEA model

$$
\begin{aligned}
&\theta^* = \min \theta \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \qquad i = 1,2,\ldots,m; \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \qquad r = 1,2,\ldots,s; \\
&\qquad \lambda_j \ge 0 \qquad j = 1,2,\ldots,n.
\end{aligned} \tag{1}
$$

where, x_{io} and y_{ro} are respectively the ith input and rth output for DMU_o under evaluation.

In order to measure allocative efficiency (AE), the following linear programming problem is employed.

$$
\begin{aligned}
&\min \sum_{i=1}^{m} p_i^o \tilde{x}_{io} \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \tilde{x}_{io} \qquad i = 1,2,\ldots,m; \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \qquad r = 1,2,\ldots,s; \\
&\qquad \lambda_j \ge 0 \qquad j = 1,2,\ldots,n.
\end{aligned} \tag{2}
$$

where p_i^o and $\tilde{x}_{io}$ respectively represent the unit price for the ith input and the cost minimization input quantity for DMU_o.

Model (2) calculates the minimum cost for DMU_o. Let $\tilde{x}_{io}^*$ be the optimal values for (2), then we define cost efficiency (CE) which measures the difference between actual and optimal costs as

$$
\mathbf{CE} = \frac{\sum_{i=1}^{m} p_i^o \tilde{x}_{io}^*}{\sum_{i=1}^{m} p_i^o x_{io}}
$$

Next AE (allocative efficiency) is defined as

AE = CE/TE

where, **TE** (technical efficiency) is the optimal value to (1), i.e., **TE** = θ^*. Obviously, **CE** $\le$ 1. If **CE** = 1, then DMU_o achieves both allocative efficiency and technical efficiency, i.e., cost minimization.

If one employs an assurance region (AR) approach to estimate the AE, ratios of the form

$$
\alpha_i \le \frac{v_i}{v_{i_o}} \le \beta_i, \ (i = 1, \ldots, m) \tag{3}
$$

are introduced into the dual linear programming model to (1). That is

$$\max \sum_{r=1}^{s} u_r y_{ro}$$
$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{s} v_i x_{ij} \leq 0 \qquad j = 1,\ldots,n; \tag{4}$$
$$\sum_{i=1}^{m} v_i x_{io} = 1 \quad and \ (3)$$
$$u_r, v_i \geq 0.$$

On the other hand, as discussed in Zhu (1996b), the following DEA/Preference Structure (DEA/PS) model may also be used to characterize AE.

$$\min \frac{\sum_{i=1}^{m} \pi_i h_i}{\sum_{i=1}^{m} \pi_i} \quad (= \sum_{i=1}^{m} \tilde{\pi}_i h_i)$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \pi_i x_{io} \qquad i = 1,2,\ldots,m; \tag{5}$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1,2,\ldots,s;$$
$$\lambda_j,\ h_i \geq 0.$$

where $\tilde{\pi}_i = \frac{\pi_i}{\sum_{i=1}^{m} \pi_i}$ and π_i (i = 1, …, m) are user-specified weights that reflect the degree of desirability of adjustments in the current input levels. Note that the input levels are allowed to either decrease or increase.

The above DEA models are developed under constant returns to scale (CRS). Similarly, one can easily develop a set of DEA models under variable returns to scale (VRS) by adding $\sum_{j=1}^{n} \lambda_j = 1$ into (1), (2) and (5). As a result, there will be a new free dual variable in (4) (see Charnes *et al.* (1994) or Cooper *et al.* (2000)).

3. RESULTS

We will apply the CRS and VRS versions of DEA models described in the previous section to estimate the AE for 33 Tennessee county jails. Two outputs – Jail Days and Average Term and three inputs – Annual Employees, Admissions and Total Square Feet are selected for the analysis (see Hayes and Millar (1990) for a complete explanation for these factors). The raw data and actual unit prices are provided in Table 1.

Model (1) and model (1) with $\sum_{j=1}^{n}\lambda_j = 1$ are used to calculate TE under CRS and VRS, respectively (see Table 2). By employing (2) and actual unit input prices, minimum cost for each jail is calculated (see Mensah and Li (1993) for a different approach to AE). Furthermore, CE and AE are derived.

In order to measure DEA/AR efficiency, based upon Table 1, the unit price ranges $212.34263 \le p_{employees} \le 520.26504$, $5.83716 \le p_{admissions} \le 238.98986$ and $1.13715 \le p_{square} \le 31.50005$ are used to develop the lower and upper bounds in (3) as follows.

$$0.8885 = \frac{212.34263}{238.98986} \le \frac{v_{employees}}{v_{admissions}} \le \frac{520.26504}{5.83716} = 89.1298$$

$$6.7410 = \frac{212.34263}{31.50005} \le \frac{v_{employees}}{v_{square}} \le \frac{520.26504}{5.83716} = 457.5166$$

$$0.1853 = \frac{5.83716}{31.50005} \le \frac{v_{a\,dimissions}}{v_{square}} \le \frac{238.98986}{1.13715} = 210.1656$$

As in Zhu (1996b), we use an average cost vector obtained from 19 VRS technically efficient jails to develop the preference weights, π_i (i = 1, .., 3), in (5),. That is,

$$\pi_i = \frac{1}{|\mathbf{E}|}\sum_{j\in\mathbf{E}} p_i^j x_{ij} \quad (i = 1, 2, 3)$$

where $\mathbf{E}$ and $|\mathbf{E}|$ represent respectively the set of and the number of VRS technically efficient jails. We normalize the preference weights: $\tilde{\pi}_1 = 0.0349$, $\tilde{\pi}_2 = 0.6691$, and $\tilde{\pi}_3 = 0.2960$.

Table 2 reports the CE, AE, DEA/AR and DEA/PS scores under CRS and VRS, respectively.

A paired-difference t-test is applied to each two of the above four scores. The results of the t-test show that in both CRS and VRS cases, the mean of the paired differences between the AE and DEA/AR (DEA/PS) scores are significantly greater than zero (see Table 3). The Pearson product-moment correction indicates a higher correlation between CE and DEA/AR (DEA/PS). This result implies that DEA/AR and DEA/PS may be more closely related to as rather than **AE**.

Table 4 reports the performance rankings by **CE**, **AE**, DEA/AR and DEA/PS. The following hypotheses are to be tested by the Spearman rank correlation coefficient[1].

Hypotheses

A: $\mathbf{H_o}$: **AE** rankings and DEA/AR rankings are independent;
$\mathbf{H_1}$: **AE** rankings and DEA/AR rankings are directly related.

B: $\mathbf{H_o}$: **AE** rankings and DEA/PS rankings are independent;
$\mathbf{H_1}$: **AE** rankings and DEA/PS rankings are directly related.

C: $\mathbf{H_o}$: **CE** rankings and DEA/AR rankings are independent;
$\mathbf{H_1}$: **CE** rankings and DEA/AR rankings are directly related.

[1] Such technique has also been used in other DEA related studies (Zhu (1998)). We do not use other DEA-rank techniques (Sinuany-Stern and Friedman (1998), Friedman and Sinuany-Stern (1998) and Sueyoshi (1999)) to further analyze the efficiency scores, since such task is beyond the scope of the current paper.

D: H_o: **CE** rankings and DEA/PS rankings are independent;
H_1: **CE** rankings and DEA/PS rankings are directly related.

The Spearman rank correlation coefficients in Table 3 show that we can not reject H_o for A and B. Thus, each of the non-parametric statistical tests indicates that CE is more directly related to the DEA/AR and DEA/PS scores than AE is.

The following development shows that DEA/PS can be used to obtain exact CE scores. Since the actual cost -- $\sum_{i=1}^{m} p_i^o x_{io}$ is a constant for a specific DMU_o, CE can be directly calculated from the following modified (2).

$$\min \frac{\sum_{i=1}^{m} p_i^o \tilde{x}_{io}}{\sum_{i=1}^{m} p_i^o x_{io}}$$

$$\begin{aligned} s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \tilde{x}_{io} \quad & i = 1,2,\ldots,m; \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad & r = 1,2,\ldots,s; \\ & \lambda_j \ge 0 \quad & j = 1,2,\ldots,n. \end{aligned} \tag{6}$$

Now let $\tilde{x}_{io} = h_i x_{io}$, ($h_i \ge 0$), then (6) is equivalent to the DEA/PS model (5) with $\pi_i = p_i^o x_{io}$. This indicates that if one imposes a proper set of preference weights for each DMU under consideration, then the DEA/PS model (5) yields **CE**.

4. CONCLUSIONS

The paper examines consistency and inconsistency between the allocative efficiency (AE) derived from prices information and the DEA efficiency models incorporating value judgment. Specifically, we examine the relationships among AE, CE, DEA/AR and DEA/PS scores. It is shown that both DEA/AR and DEA/PS scores are more strongly correlated with the CE scores. This result confirms the conclusion that the use of inequality relations in the form of an AR may result in a fusion of TE and AE (Sueyoshi, 1992). Finally, we shall point out that the non-parametric statistical tests indicate a higher correlation between DEA/AR and DEA/PS scores. In the absence of exact price information, DEA/AR or DEA/PS is a valuable method for estimating the overall efficiency of DMUs.

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Table 1 Raw data for Tennessee county jails

Jail No.	Inputs: Employees (Annual)	Inputs: Admissions	Inputs: Total Sq. Ft.	Output: Jail Days	Output: Average Term	Actual Unit Prices ($): Employee	Actual Unit Prices ($): Admission	Actual Unit Prices ($): Sq. Ft.
1	371	72777	71872	396389	5.44662	456.32599	31.20395	25.15954
2	260	41130	86583	271158	6.59271	335.46526	28.41425	7.30080
3	69	6728	39555	96726	14.37663	412.45917	238.98986	2.79668
4	79	9029	10224	66922	7.4119	420.00896	88.39836	8.82942
5	10	2379	14552	27429	11.52963	296.84996	31.84397	3.74604
6	15	3600	7616	34390	9.55278	458.17472	31.99749	6.90572
7	9	4538	3998	30732	6.77215	443.87613	21.46981	9.73012
8	19	3497	8537	29461	8.42465	375.03423	38.03094	4.04200
9	15	2731	3204	25185	9.2219	298.55297	30.08209	12.73297
10	4	5044	3757	18347	3.63739	376.46947	10.60958	2.40565
11	8	894	7304	9437	10.55593	221.18108	55.16758	1.13715
12	7	2530	4013	19052	7.53043	456.49099	29.62562	8.86438
13	7	3650	3525	22362	6.12658	319.76706	27.42326	8.41690
14	7	2230	2427	21471	9.62825	316.49945	22.06806	6.89471
15	9	2562	4226	10582	4.13037	231.42000	29.36931	8.15366
16	9	2202	2864	12786	5.80654	305.52656	29.43852	27.64109
17	5	1571	1290	12829	8.16614	268.37262	40.34826	12.72998
18	5	2238	2960	17024	7.60679	471.50146	54.09195	12.12857
19	7	1023	7712	16505	16.13392	228.56060	83.61645	3.15528
20	8	2130	4736	16019	7.52066	241.71168	45.46121	13.82640
21	4	1001	2896	7394	7.38661	314.96284	36.14260	3.15227
22	6.2	2862	2850	9855	3.4434	520.26504	17.74436	8.40709
23	8	1718	2408	9337	5.43481	212.34236	30.82676	5.45439
24	4	3278	2778	11275	3.4396	235.32673	9.79344	2.45044
25	4	519	1908	5765	11.1079	337.28178	44.84200	4.65881
26	5	1892	2960	8046	4.25264	324.36165	17.20160	5.68869
27	3	6074	3914	8352	1.37504	246.64936	5.83716	1.87692
28	5	624	1125	6540	10.48077	215.67004	47.61849	10.19608
29	4	413	636	4398	10.64891	266.43034	73.95034	31.50005
30	4	554	5712	4841	8.73827	312.17269	58.26300	1.14579
31	3	988	1899	4631	4.68725	226.29051	23.02530	4.00685
32	5	554	598	3085	5.56859	273.63685	52.23392	3.54569
33	4	302	1456	1374	4.54967	268.59707	40.68212	4.53777

Source: Hayes and Millar (1990).

Table 2 Efficiency scores

	Constant Returns to Scale (CRS)					Variable Returns to Scale (VRS)				
Jail No.	TE	CE	AE	DEA/AR	DEA/PS	TE	CE	AE	DEA/AR	DEA/PS
1	0.600	0.425	0.708	0.595	0.625	1.000	1.000	1.000	1.000	1.000
2	0.606	0.405	0.669	0.606	0.575	1.000	0.944	0.944	1.000	0.967
3	0.984	0.763	0.776	0.982	0.955	1.000	1.000	1.000	1.000	1.000
4	0.762	0.443	0.581	0.757	0.744	1.000	0.620	0.620	1.000	1.000
5	1.000	0.844	0.844	0.822	0.779	1.000	1.000	1.000	0.862	0.805
6	0.892	0.777	0.871	0.892	0.841	1.000	1.000	1.000	1.000	0.941
7	1.000	0.970	0.970	0.827	0.767	1.000	1.000	1.000	1.000	0.914
8	0.757	0.589	0.778	0.757	0.719	0.768	0.660	0.860	0.768	0.764
9	0.939	0.681	0.725	0.938	0.923	1.000	0.750	0.750	1.000	0.996
10	1.000	1.000	1.000	0.509	0.468	1.000	1.000	1.000	0.523	0.473
11	0.687	0.599	0.871	0.682	0.646	0.746	0.671	0.900	0.736	0.667
12	0.856	0.814	0.951	0.761	0.713	0.860	0.833	0.969	0.765	0.715
13	0.923	0.838	0.908	0.709	0.674	0.928	0.865	0.932	0.730	0.692
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	0.405	0.383	0.946	0.405	0.384	0.496	0.473	0.953	0.424	0.389
16	0.586	0.507	0.866	0.586	0.569	0.599	0.561	0.937	0.589	0.575
17	1.000	1.000	1.000	1.000	0.938	1.000	1.000	1.000	1.000	0.945
18	1.000	1.000	1.000	0.809	0.760	1.000	1.000	1.000	0.826	0.765
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	0.702	0.637	0.907	0.699	0.659	0.744	0.672	0.903	0.707	0.661
21	0.959	0.954	0.995	0.671	0.623	0.966	0.954	0.987	0.700	0.626
22	0.476	0.474	0.995	0.388	0.373	0.646	0.621	0.961	0.422	0.386
23	0.540	0.474	0.878	0.540	0.522	0.583	0.576	0.988	0.551	0.529
24	0.805	0.782	0.971	0.440	0.407	0.920	0.863	0.938	0.462	0.424
25	1.000	1.000	1.000	0.922	0.873	1.000	1.000	1.000	0.928	0.881
26	0.531	0.516	0.972	0.435	0.406	0.715	0.712	0.995	0.485	0.418
27	0.809	0.656	0.811	0.213	0.199	1.000	0.789	0.789	0.235	0.226
28	0.975	0.907	0.930	0.975	0.944	0.981	0.909	0.927	0.980	0.944
29	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
30	0.798	0.783	0.981	0.682	0.570	0.962	0.889	0.924	0.760	0.608
31	0.916	0.880	0.961	0.481	0.431	1.000	1.000	1.000	0.570	0.446
32	0.668	0.457	0.685	0.645	0.583	1.000	0.778	0.778	1.000	0.842
33	0.584	0.432	0.739	0.574	0.461	1.000	1.000	1.000	1.000	1.000
Average	0.811	0.727	0.888	0.706	0.671	0.906	0.853	0.941	0.789	0.745

Table 3 Correlations among different measures

	t-test	Pearson Product-Moment Correlation	Spearman Rank-Order Correlation
Constant Returns to Scale (CRS)			
AE vs. DEA/AR	t-statistic = 4.458	0.077	0.210
AE vs. DEA/PS	t-statistic = 5.243	0.062	0.158
CE vs. DEA/AR	t-statistic = 0.597	0.569	0.560
CE vs. DEA/PS	t-statistic = 1.563	0.547	0.564
DEA/AR vs. DEA/PS	t-statistic = 7.132	0.991	0.983
Variable Returns to Scale (VRS)			
AE vs. DEA/AR	t-statistic = 3.592	0.005	0.309
AE vs. DEA/PS	t-statistic = 4.461	-0.003	0.319
CE vs. DEA/AR	t-statistic = 1.844	0.521	0.548
CE vs. DEA/PS	t-statistic = 2.909	0.486	0.534
DEA/AR vs. DEA/PS	t-statistic = 5.947	0.984	0.960

Table 4 Rank

	Constant Returns to Scale (CRS)				Variable Returns to Scale (VRS)			
Jail No.	CE	AE	DEA/AR	DEA/PS	CE	AE	DEA/AR	DEA/PS
1	31	30	23	19	7.5	7.5	7	4
2	32	32	22	22	16	21	7	9
3	18	27	5	4	7.5	7.5	7	4
4	29	33	15	13	30	33	7	4
5	12	24	11	10	7.5	7.5	16	16
6	17	22	9	9	7.5	7.5	7	12
7	8	13	10	11	7.5	7.5	7	13
8	23	26	14	14	28	29	18	18
9	19	29	7	7	24	32	7	8
10	4	4	27	26	7.5	7.5	28	27
11	22	21	19	18	27	28	21	21
12	14	15	13	15	21	18	19	19
13	13	18	16	16	19	24	22	20
14	4	4	2.5	2	7.5	7.5	7	4
15	33	16	31	31	33	20	31	31
16	25	23	24	24	32	23	25	25
17	4	4	2.5	9	7.5	7.5	7	10
18	4	4	12	12	7.5	7.5	17	17
19	4	4	2.5	2	7.5	7.5	7	4
20	21	19	17	17	26	27	23	22
21	9	9	20	20	15	17	24	23
22	26.5	8	32	32	29	19	32	32
23	26.5	20	26	25	31	16	27	26
24	16	12	29	29	20	22	30	29
25	4	4	8	8	7.5	7.5	15	14
26	24	11	30	30	25	15	29	30
27	20	25	33	33	22	30	33	33
28	10	17	6	5	17	25	14	11
29	4	4	2.5	2	7.5	7.5	7	4
30	15	10	18	23	18	26	20	24
31	11	14	28	28	7.5	7.5	26	28
32	28	31	21	21	23	31	7	15
33	30	28	25	27	7.5	7.5	7	4