CONSTANTINE PORPHYROGENETUS INTERNATIONAL ASSOCIATION

Journal of Management Sciences and Regional Development
Issue 5, July 2005
Correspondence: ikarkazis@aegean.gr

http://www.stt.aegean.gr/geopolab/GEOPOL%20PROFILE.htm
ISSN 1107-9819
Editor-in-Chief: John Karkazis

MINIMUM COST FACILITY ALLOCATION WITH AN EQUITY RESTRICTION

Glykeria Kalfakakou
Department of Civil Engineering
Aristotle University of Thessaloniki
Thessaloniki, Greece

Abstract. The concept of equity in facility location problems can in general be interpreted as the attempt to equalize the quality of the service for all demands, since the perception of equity in facility location problems contributes much to the economic and social development of populations/communities dispersed over a certain geographical area. Such problems have many variations depending on the nature of the data as well as on the objectives. In this paper a new method is developed for locating facilities at minimum cost under a predetermined budget restriction while, at the same time taking into account the factor of distribution time equity in conjunction with a maximum acceptable delivery time. A measure is used related to the concept of equity that is expressed by a ratio which connects the closest and the longest travel time in a supply-demands system.

The method developed here has been implemented on a computer program and a sample computational experiment is given. Finally the last Section is devoted to the conclusions.

Keywords: Equity, Facility Location, Depth First Search, Optimization

1. INTRODUCTION

Facility Location Problems (FLP's) concerning the supply of given demands have a significant role in network design analysis since they contribute greatly to both the economic and social development of communities dispersed over a certain geographical area. Therefore, a significant amount of research has been performed and many variations of such problems have been stated depending on the diverse type of the supply–demand system, such as the allocation of fire stations, warehouses, bank sites, stay points for ambulance or police patrol cars, retail facilities, see ReVelle and Eiselt for a notable and extensive synthesis and survey.

The optimization criteria and the restriction relations that connect the involved parameters are the main items differentiate FLP's (installation costs, prompt service fulfillment, transportation costs, etc).

The concept of equity can, in general, be interpreted as the attempt to equate the quality of the service for all the demands. In the case of the allocation of facilities, the main issue related to equity is the achievement of a near-uniform time distance in a supply-demand system. Both the importance and the impact of equity conception is referred in Mandell (1991), McAllister (1994) and Savas (1978). Nevertheless the amount of academic research does not comply with the great importance of equity in regional planning associated with direct and indirect positive influences of equity on the economic and social development of populations dispersed over a geographical area. The problem studied in this paper consists of detecting a set of facility centers in prospective locations within a predetermined budget that satisfies an equity restriction concerning the travel time that does not exceed a given value T to standard demand sites. It is assumed that the installation cost of a facility depends exclusively on its geographical location and that the service capacity of every facility is unlimited.

The given data are

- The network that comprises the demand nodes, the nodes which are candidates for the installation of a supply center and the time distance between the prospective supplies and standard demand nodes.
- The installation cost of each candidate supply, the budget restriction, say B and the maximum permitted accessibility time, say T
- A value $r = {}^{d\min}/_{d\max} \leq 1$ that indicates the measure of equity used here and which expresses a relation connecting the smallest and the greatest travel time d_{min} and d_{max} between the set of located supplies to a demand location, r is an item of proportionality between d_{min} and d_{max}.

A feasible solution must satisfy the relations $d_{max} \leq T$ and ${}^{d\min}/_{d\max} \geq r$.

Due to the fact that graph terminology and notations have not yet been unified, and for reasons of self-reliance, The next Section provides, the definitions and notations of the terms used in the subsequent pages. A formal statement of the problem dealt with here is given in Section 3, while Section 4 is dedicated to the devised procedure for solving the problem stated in the previous sections. Finally the last Section presents a sample computational experiment as well as the conclusions.

2. PRELIMINARIES – NOTATIONS

Let $V = \{v_1, v_2, \ldots, v_n\}$ be a nonempty set and $E \subseteq V \times V$ a subset of unordered couples (v_i, v_j), $v_i, v_j \in V$. The ordered pair (V, E) defines a graph $G = (V, E)$, see Harary (1969). The elements of V are usually called vertices, nodes or points and the elements of E links, lines or edges. If the lines of E have an orientation the graph is called a directed graph and the elements of E are called arcs. In this paper only traveling directions from the supplies to the demands are used, in the following, the term graph without loss of generality will be used since in an urban network the links connecting the locations have two opposite directions. A graph can easily be drawn on the plane providing a good image of the connection structure of the elements of V for relatively small graphs.

Two nodes v_i and v_j are adjacent if they define an arc, i.e., $(v_i, v_j) \in E$. The set of adjacent nodes of a node $v_i \in V$ is denoted by $\Gamma(v_i)$, i.e., $\Gamma(v_i) = \{\, y$ such that $(v_i, y) \in E \,\}$ while the nonadjacent nodes of v_i are symbolized by $\overline{\Gamma}(v_i) = V - \Gamma(v_i)$. The degree $deg(v_i)$ of a node $v_i \in V$ expresses the number of adjacent nodes to v_i, obviously $deg(v_i) = |\,\Gamma(v_i)\,|$. An n node graph G_n^c is *complete* if every pair of its nodes is adjacent, therefore for every $v \in G_n^c$ we have $deg(v) = n-1$ and the total number of arcs in G_n^c is $(n(n-1))/2$. The density d of graph G with n nodes is defined here to be the quotient of the number e of arcs in G over the number of arcs in G_n^c, thus the density of G_n^c is $d = 2e/(n(n-1))$, The set that contains all adjacent nodes of all nodes in a set $S \subset V$ is the set $\Gamma(S) = \bigcup_{v_i \in S} \Gamma(v_i)$.

A graph $G' = (V', E')$ is a subgraph of $G = (V, E)$ if $V' \subset V$ and E' contains only the arcs of E that are innovated by nodes in V' i.e, $E' = \{ (x, y) \in E$ and $x, y \in V' \}$.

A subset $S \subset V$ is a *dominating set* of $G = (V, E)$ if for every $y \in \overline{\Gamma}(S) = V - \Gamma(S)$ there exists at least a node $x \in S$ such that $(x\ , y) \in E \}$.

A *network* N is a graph to with weights assigned to its nodes and / or arcs. By I_q we denote the set of consecutive natural numbers 1, 2, ..., q. For a pair $x_i, x_j \in V$, a sequence $(x_{i_1}, x_{i_2}, \ldots x_{i_k})$ with $x_{i_k} = x_i$ and $x_{i_k} = x_j$ is called a *route* if $(x_{i_m}, x_{i_{m+1}}) \in E$ for every $m \in I_{K-1}$. A route where every node occurs only once is called a *simple* route, here only simple routes are considered. In general, there may exist more than one route that joins nodes $x, y \in V$ in a network. Therefore, we denote by $\Delta_i(x, y), i = 1,2,\ldots q(x, y)$ such a route, where $q(x\ , y)$ is the number of distinct routes that join x and y. A graph is said to be *connected* if for every pair $x\ , y \in V$ there exists at least one route $\Delta_\iota(x, y)$ and *strongly connected* if in addition there exists a route $\Delta_\iota(y, x)$. Since here we are dealing with urban networks, the corresponding graphs are strongly connected.

To every arc $(x,y) \in E$ of network N we associate a value $c(x,y)$ that expresses the duration of a vehicle to traverse (x,y). The cost $K(\Delta_i(x,y))$ of a route $\Delta_i(x,y)$ is the sum of the values of its arcs. A route with the minimum cost is a shortest route and the corresponding value is the *distance* $d(x,y)$, namely

$$d(x,y) = \min_{i \in I_{q(x,y)}} \{K(\Delta_i(x,y))\}$$

3. PROBLEM STATEMENT

Let $G=(V, A)$ be a graph associated with the given geographical network. In order to simplify the process regarding the maximum allowable travel time T from a facility to a demand node, we modify the set of arcs A to the set E that contain only arcs joining the elements of Q to the elements of D the travel distance of which have values not exceeding T, specifically, $E=\{(x,y)\ x \in Q,\ y \in D$, such that $d(x,y) \leq \mathrm{T}\}$, where $d(x,y)$ expresses the shortest path connecting x and y. In the sequel we will work with graph $G = (V, E)$ without considering value T, since the restriction of maximum travel time supply completion will always be satisfied.

The nodes that correspond to the locations of the candidate supply centers are the elements of set $Q = \{q_1, q_2, \ldots, q_z\} \subseteq V$ while $D = \{d_1, d_2, \ldots, d_m\} \subseteq V$ is the set that represents the nodes of the demand locations. To every element of $q_i \in Q$ corresponds an installation cost c_j of set $C = \{c_1, c_2, \ldots, c_z\}$.

Let $S \subseteq Q$, by $K(S)$ we denote the sum of the corresponding costs of the nodes in S, namely $K(S) = \sum_{j \in S} c_j$.

A candidate supply center might also be a demand node, thus in general we have $D \cap Q \neq \varnothing$. We say that a set $S_D \subset Q$ is a dominating set on D if $D \subseteq \Gamma(S_D)$. Consider the smallest and the greatest travel time $d_{min}(S_D)$ and $d_{max}(S_D)$ respectively of a facility in S_D to a demand location, namely,

$d_{max}(S_D)=\max\{d(x,y)\ x \in S_D,\ y \in D\}$, $d_{min}(S_D)=\min\{d(x,y)\ x \in S_D,\ y \in D\}$.

A domination set S_D^r is feasible in accordance with r if the relation $r \geq d_{\min}(s_D^r) / d_{\max}(s_D^r)$ is verified. The determination of a feasible set S_D^r belongs to the class of *NP-complete* problems. Let $S(r,D)$ represent the family of feasible sets S_D^r. A solution to our problem is a set S_D^{*r} such that $K(S_D^{*r}) = \min\{K(S_D^r) \leq B,\ S_D^r \in S(r,D)\}$.The detection of an optimal set S_D^{*r} is clearly an *NP- hard problem,* see Garey and Johnson (1979).

4. SOLUTION PROCEDURE

The methodology used to solve the problem in question is a depth first backtracking scheme that produces successively feasible subsets S_D^r with smaller values of installation costs $K(S_D^r)$ in every stage. Next, an interpretation of some notations used in the sequel are given before stating the operations of the corresponding procedure in a QuickBasic-like presentation.

S_k : A stack that contains the nodes of the investigated k nodes path of the working tree.

A_m : Family of m alternative best solution S_D^r found so far in an instant of the procedure.

BST : Smaller value $K(S_k)$ of the feasible sets examined so far in an instance of the process.

WC : Expresses the value of $K(S_k)$.

Read data $\{V, A, C, Q, D, \mathrm{T}, \mathrm{r}, B\}$

"Preprocessing Operations"

Transform the arc set A to E so that $E \leftarrow \{(x,y)\ x \in Q,\ y \in D \text{ and } d(x,y)\} \leq T\}$.

Reorder the elements of $q_i \in Q$ so that the associated costs $\{c_i, c_j\}$ for every pair $\{q_i, q_j\}$ verify the relation $c_i \geq c_j$, for $i \geq j$

" Initial Conditions "

BST $\leftarrow$ B, WC $\leftarrow$ 0 , $A_0 \leftarrow \varnothing$, S $\leftarrow \varnothing$, $Q_0 \leftarrow Q$, $LR \leftarrow$ false

" Main Procedure "

While LG

While BOOL and $Q_k \neq \varnothing$

Set $k \leftarrow k+1, S_k \leftarrow S_{k-1} + \{q_k\}$, $q_k \in Q_{k-1}$ $\rightarrow$ " Branch"

Set $WC \leftarrow WC + c_k$, $q_k \in Q_{k-1+}$, $Q_k \leftarrow Q_{k-1} - \{q_k\}$.

```
If WC ≤ BST                                →        " Optimality test "
        Then
        If V – S_k ⊆ Γ( S_k )              →        " Domination test "
         Then Call DISTR
           If FLG Then Call Store and Set BOOL← false
           ENDIF
        ENDIF
  ENDIF
 WEND { BOOL }
Set k← k-2 , If k < 0 Then Set LG ← false        →        " Backtrack "
             Else Set S_k ← S_k+2 - { q_k+1 , q_k+2}, Q_k ← Q_k -{ q_k+1 }
and
             Set WC ← WC - c_k+1 - c_k+2 .
WEND { LG }
 If LR Then Write results Else Write " There is no Solution for the
given data"
       End
```

```
SUB DIST
   Set FLG ← false
 For every v∈V-S_k set
 D (v) ← min { d(y,v) such that y ∈ S_k
 For every v∈ V- S_k set
 r1 ← min { D(v) } , r2 ← max { D(v) }
   ratio ← r1 / r2 , If ratio ≥ r Then Set FLG← true  →
                                          " Feasibility test "
 Endsub
```

Subroutine Store

LR = True → " Solution occurrence"

If $WC < BST$ then Set $m \leftarrow 0$, $BST \leftarrow WC$

Else Set $m \leftarrow m + 1$, $A_m \leftarrow S_k$ → " Alternative Solution"

Endif

Endsub

A set Q_k is associated with every stack S_k , Q_k is successively reduced during the algorithmic execution since it contains the nodes that have not yet been used for augmenting the corresponding S_k.

The backtrack operation is performed two levels backwards on the working tree because a one tree level backtracking will lead to a branching that will result in a greater value of *WC*, this is due to the fact that the elements of Q are positioned in increasing order of that the density of a graph can be defined their installation cost in conjunction with the lexicographical branching on the elements of Q_k.

5. COMPUTATIONAL EXPERIMENT

In Section 2, it was seen as the number of its links over all possible pairs of nodes(all possible links). In the case of a Supply Demand System (SDS) we are not concerned with links that connect the demands to each other.

A complete connection structure of a SDS occurs when every element of Q , with $z = | Q |$ is joined with every element of D, with $m = | D |$, thus a complete connection structure comprises m $\times$z links.

Clearly every demand must be linked with at least one facility, therefore the minimum number of links among the supplies and demands is m = | D | , namely, when every demand is connected with exactly one facility. The density π of a SDS is defined as the quotient of its number of links over the total number of links of a complete connection structure SDS that comprises the same number of demands and candidate supply nodes, thereafter the density π must satisfy the relation $1 / z \leq \pi \leq 1$. It is easy to verify that in the extreme case where $\pi = 1$ the solution to our problem is the facility with the minimum installation cost.

Most equity measures use the gap that separates distances between supplies and demands, (see, Eiselt et al.,1993). The main goal of the problem stated here is the determination of a set of facility locations with the minimum installation cost that satisfy all the demands in conjunction with the consideration of an equity constraint stipulated as a relation of proportionality between the greater and the smaller distance in the resulting solution.

The solution method described in section 4 has been coded in Fortran 77 and executed on a Pentium V with a 3.2 processor. The corresponding program was tested on randomly generated networks with the use of a uniform distribution. The range of the facilities installation costs, the distance among the candidate facilities sites and the demand locations were selected from the intervals [10, 25] and [10, 50] respectively, also 70% of the sum of the supplies installation cost was given to the budget B ,namely $c_i \in C, c_i \in [10,50]$ and $\forall\ x \in Q, y \in D \Rightarrow d(x,y) \in [10,25]$ and $B = \sum_{c_i \in C} c_i . 0.7$.

The program running tests were performed on supply-demand systems with 50,100,150 demand locations and with 15,20,25 candidate supply sites.

Table 1 gives the execution time in seconds and in parenthesis the number of facility nodes in the optimal solution that resulted after the application of the program on networks with density $\pi = 0.8$ and with equity ratio restriction $r = 0.6$.

N

NQ

		0	0
	4)	(7)	(5)
	4)	109 (5)	5 (7)
	4)	(5)	9 (7)

Table 1.

Some results seem to be unusual. For example, the execution time consumed for problems of a larger size is less than for problems for a smaller size, of course this is not a case that happens frequently, however it is a notable characteristic of *NP- hard* problems.

The existence of a feasible solution depends on the data as well on their relations, the possibility of obtaining a solution increases when the density of the SDS is relatively large because in this case the cardinality of the family of the dominating sets of the network is correspondingly large, on the contrary the chance of getting a solution is greater when the equity ratio is smaller. The data that correspond to the results of table 1 were selected in order to obtain solutions as well as to depict the *NP-hardness* of the problem posed in this paper.

6. CONCLUSIONS

The previous sections a describes a method for the determination of a set of facility locations so to satisfy an equity condition taking simultaneously into account the factors of a budget limitation and of a given maximum acceptable time for the fulfilment of a required demand in a distribution network.

In the case where a particular data set does not lead to a solution, the decision maker might either suitably increase the maximum delivery time and/or reduce the equity ratio restriction.

Since the concept of equity emerged from the factor of fairness or justice it is evident to consider the notion of gravity distribution. A weight is associated to every demand node which might represent the corresponding population size or the frequency of the demand requirements. This situation can be faced by partitioning the demand nodes into gravity classes C_z, z=1,2…., according to their weights and to correspond a value, say g_z to every gravity class C_z. The relation $g_i > g_j$ means that the weight of every node in class C_i is greater than the weight of any node in class C_j. The consideration of the gravity concept is resolved if the distance $d(v_i, v_j)$ is replaced by the value $d(v_i, v_j).g_k$, where $v_j \in C_k$.

Furthermore the modification on the arc set A mentioned in the first part of Section 3 of the primary graph $G=(V, A)$ to the set E is performed so that

$$E=\{(x,y)\ x\in Q, y\in D, \text{ such that } (d(x,y).\ g_k) \leq \mathrm{T}, \text{ where } y\in C_k\ \}.$$

The application of the method may give alternative solutions, in that case we can use additional criteria, for example we may select the solution that correspond to the greater value of the equity measure $d_{\min}/d_{\max} > r$.

The equity condition as it is presented here is mostly applicable to public branches (fire station, police patrol, ambulance stations, administration authorities, etc) and in some cases it might meet real-life situations in private organisations such as the installation of warehouses that stores sensible goods. Finally In the case of unwelcome facilities or hazardous materials, see, Erkut et al,1989 and Karkazis et al, 1995. it is evident that the equity constraint is converted to $d_{\min}/d_{\max} \le r$.

REFERENCES

1. **Eiselt, H.A , Laporte, G. and Thisse, J.-F (1993).** "Competitive location models: A framework and bibliography", *Transportation Science* 27, pp. 44-54.

2. **Erkut, E.,(1993)** , " Inequality measures for location problems", *Location Science* 1, pp. 199-217.

3. **Erkut, E. and Newman, S.,(1989)** , "Analytical models for locating undesirable facilities", *European Journal of Operational Research* 40, pp. 275-279.

4. **Garey, M.R. and Johnson D.S. (1979),** *"Computers and Intractability: A Guide to the theory of NP-Completeness"*, Freeman, San Francisco.

5. **Harary F., (1969),** *"Graph theory"*, Addison Wesley.

6. **Karkazis, J. and Boffey TB (1995)**, "Optimal location of routes for vehicles transporting hazardous materials", *European Journal of Operational Research*, Vol 86, Issue 2 pp 201-215.

7. **McAllister, D.M. (1976)**, "Equity and efficiency in public facility location", *Geographical Analysis* 8, 47-63.

8. **Mandell , M., (1991)**, "Modeling effectiveness-equity tradeoffs in public service delivery systems", *Management Science* 37, pp 467-482.

9. **Marsh, M.T and Schiling, D.A., (1994)**, "Equity measurement in facility location analysis: A review and framework", European Journal of Operational Research 74, pp. 1-17.

10. **ReVelle C.S and Eiselt H.A,(2005)**, *"Location analysis : A synthesis and survey"*, 165, pp 1-19.

11. **Savas, E.S. (1978)**, "On equity in providing public services", *Management Science* 24/3, pp.800-808.